Shape Sensitivity Analysis of Flutter Response of a Laminated Wing

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Abstract

An approach is presented for calculating the shape sensitivity of a wing flutter response with respect to changes in geometric shape. Three methods are used to calculate the sensitivity of the flutter eigenvalue. The first method is purely a finite-difference calculation of the eigenvalue derivative directly from the solution of the flutter problem for two different values of the shape parameters. The second method uses an analytic expression for the eigenvalue sensitivities of a general complex matrix, where the derivatives of the aerodynamic, mass, and stiffness matrices are computed using a finite-difference approximation. The third method also uses the same analytic expression for the eigenvalue sensitivities, but the sensitivity of the aerodynamic matrix is computed analytically. In each method, a modified strip aerodynamic theory and an equivalent plate structural analysis technique (to represent the wing structure) are used to predict the dynamic aeroelastic behavior of the wing. Results from applying all three methods to a sample problem are found to be in good agreement with each other.

Contents

Flutter, an aeroelastic instability, is a self-sustaining oscillation that involves a coupling between inertial, elastic, and aerodynamic forces. Flutter analysis capabilities have been available for well over four decades. Yates[1] developed a modified strip analysis to analyze rapidly the flutter characteristics for finite-span swept and unswept wings at subsonic and supersonic speeds. This method is still used occasionally today to calculate the lift and moment forces. Since it would be advantageous to the preliminary designer to have a tool that can be used to predict rapidly the changes in flutter response with the changes in basic shape parameters, the present approach based on modified strip analysis is presented.

Adelman and Haftka[2] have shown that structural sensitivity analysis has been available for over two decades and has become a versatile design tool, rather than just an instrument of optimization programs. Structural sensitivity analysis has seen much use in airframe sizing under aeroelastic constraints. For example, Rudisill and Bhatia[3] developed expressions for the analytical derivatives of the eigenvalues, reduced frequency, and flutter speed with respect to structural parameters for use in minimizing the total mass. Pedersen and Seyranian[4] examined the change in flutter speed as a function of change in stiffness, mass, boundary conditions, or load distribution. In contrast, the aerodynamic sensitivity analysis of a wing aeroelastic response with respect to shape parameters has not been available.

Barthelemy and Bergen[5] explored the analytical shape sensitivity derivatives of a wing static aeroelastic response. This study included exact sensitivity derivatives of the aerodynamic influence coefficients. These authors found that the second derivatives of the wing's aeroelastic characteristics, such as section lift, angle of attack, rolling moment, induced drag, and divergence dynamic pressure, for subsonic subcritical flow, with respect to geometric parameters are small enough that these characteristics can be approximated by sensitivity-based linear approximations. The present work is an extension of the work performed by Barthelemy and Bergen[5] and details the theoretical and computational derivation of a method to obtain the sensitivity of a wing flutter response to changes in its geometry. Specifically, the objective is to determine the derivatives of flutter speed and frequency with respect to wing area, aspect ratio, taper ratio, and sweep angle.

Analysis

The traditional V-g method flutter analysis leads to the following eigenvalue equation at the airspeed associated with neutral stability (flutter):

$$\Big[[I]\lambda - [B]\Big]\Big\{C\Big\} = \Big\{0\Big\} \tag{1}$$

where B is a generalized complex matrix, λ the eigenvalue, and $\{C\}$ the eigenvector. The eigenvalue λ is defined as $\lambda = (1 + ig)/\omega^2$, and the generalized complex matrix $[B]$ is defined as

$$[B] = [K]^{-1}[M + A] \tag{2}$$

where $[K]$ and $[M]$ are the stiffness and mass matrix, respectively, and g is the structural damping coefficient introduced in the V-g method. Since the aerodynamic formulation restricts the chordwise sections to remain straight during oscillations, only high aspect ratio wings will be analyzed in this study, and only bending and torsional deformations are considered.

This study focuses on the calculation of the sensitivity of the flutter speed, flutter frequency, and reduced frequency to geometric shape parameters, namely: 1) aspect ratio, 2) surface area, 3) taper ratio, and 4) sweep angle. To compute these sensitivities, a method for calculating the desired derivatives using analytically obtained sensitivities of the aerodynamic

Received Nov. 14, 1988; presented at the AIAA/ASME/ASCE/AHS/ASC 30th Structural Dynamics and Materials Conference, Mobile, AL., April 1989; synoptic received Oct. 3, 1989. Full paper available at AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, $4.00; hard copy, $9.00. **Remittance must accompany order.**

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Table 1 Comparison of eigenvalue derivatives with respect to four parameters

	Case 1: finite-difference	Case 2: semianalytic	Case 3: analytic
λ	2.8831 E-5	2.4199 E-5	2.6761 E-5
S	2.4151 E-5	2.4593 E-5	2.4571 E-5
Æ	7.8769 E-6	8.1694 E-6	8.1562 E-6
tp	2.6719 E-4	2.6571 E-4	2.6330 E-4

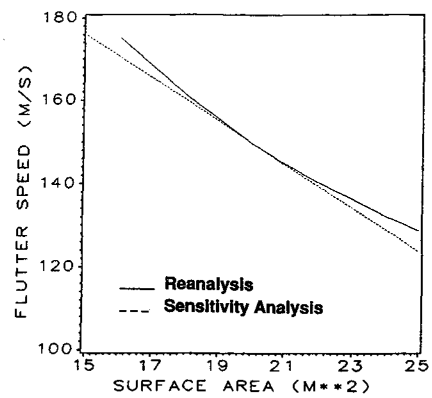

Fig. 1 Variation of flutter speed with surface area.

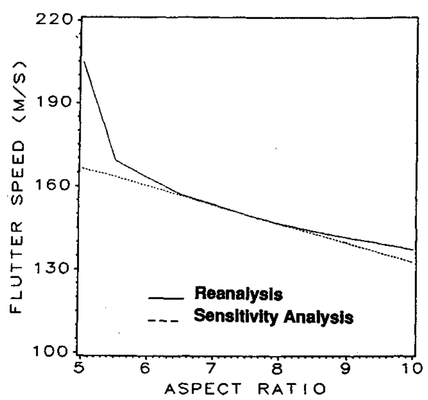

Fig. 2 Variation of flutter speed with aspect ratio.

matrix was developed and validated. This validation was accomplished by calculating the eigenvalue derivatives using three different methods.

In the first method of calculating the derivatives, the flutter problem is solved twice and the derivatives of the eigenvalues are approximated using a forward finite-difference scheme. The second and third methods use the expression for the eigenvalue derivative that is derived using the main and adjoint problem. The eigenvalue derivative in terms of flutter frequency, damping, and their derivatives can be easily written. In both the second and third methods, the derivatives of the mass matrix and the inverse of the stiffness matrix are obtained using the forward finite-difference method, but the procedure for obtaining the derivatives of the aerodynamic matrix $[A]$ with respect to a geometric shape parameter is different. In one case, they are computed by a finite-difference method; and in the other, they are derived from an analytic method. This last approach is the unique, key aspect of this work.

The calculation of the sensitivity of the aerodynamic matrix $[A]$ is made difficult by the fact that this matrix depends on shape parameters p_s and also the reduced frequency k_n. The reduced frequency is not really an independent variable, as its value for a new value of $p_s(=p_s^{\text{old}}+\Delta p_s)$ should be such that the imaginary part of the eigenvalue corresponding to the perturbed configuration should be zero. To obtain the value of Δk_n, an iterative procedure was used.[7]

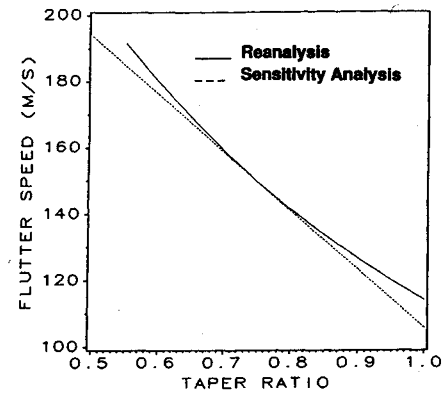

Fig. 3 Variation of flutter speed with taper ratio.

The present formulation was first evaluated by studying various examples for which alternative results are available.[7]

The "analytic" derivative of the eigenvalue with respect to various parameters, namely, the surface area S, the aspect ratio *Æ*, the taper ration tp, and sweep λ, were compared with those obtained using the two previously described methods, that is, 1) the purely finite-difference method and 2) a semianalytic approach in which the desired derivatives were obtained using a forward finite-difference scheme. The results are shown in Table 1. Reasonable agreement exists between the various sets of results. Differences that do appear in the derivatives with respect to sweep and *Æ* may be due to 1) the presence of numerical noise in the finite-difference result and 2) the noninclusion of some minor effect of sweep change on the complex eigenvalues in the other two approaches. Figures 1-3, respectively, show the variation of flutter speed with respect to three geometric shape parameters: 1) the surface area, 2) the aspect ratio, and 3) the taper ratio as obtained using the sensitivity of the flutter speed with respect to the three parameters. The results are obtained for a baseline design of $S = 20$ m, *Æ* $= 7.5$, $tp = 0.75$, and $\lambda = 0.0$. For the sake of comparison, the flutter speeds obtained by performing flutter analyses at the perturbed values are also shown in the respective figures. These are called reanalysis values because the V-g method was used to solve for the flutter speed of the perturbed value. These show excellent agreement over a range that can be useful to the designer in the initial design phase.

Acknowledgments

This research was conducted under NASA Contract NAS-1-18471-task 5.

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